fcavaibhavjain | vaibhavjainca | vaibhavjain_ca

VALUATIONS - FORECASTING

Let's talk business beyond the literature

Vaibhav Jain

B.Com(Hons), FCA, ACS, LLB, MBF (ICAI), FAFD (ICAI), Registered Valuer(S&FA), Certified Independent Director (IICA)

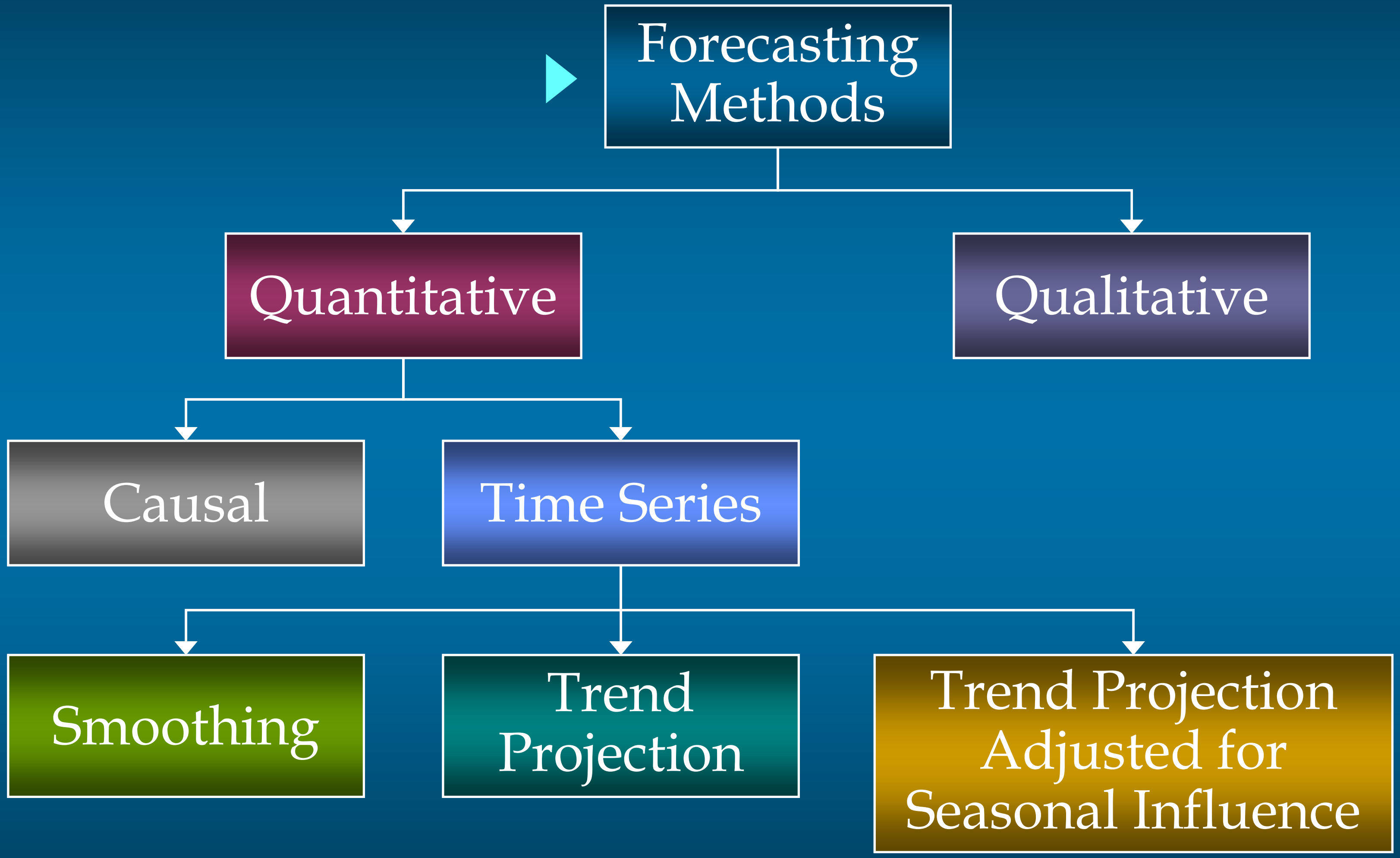
Mobile: +91.9711310004 | Email: vaibhavjain@inmacs.com

New Delhi | Gurgaon | Noida | Chandigarh | Mumbai

Forecasting

- Quantitative Approaches to Forecasting
- Components of a Time Series
- Measures of Forecast Accuracy
- Smoothing Methods
- Trend Projection
- Trend and Seasonal Components
- Regression Analysis
- Qualitative Approaches

Forecasting Methods



Quantitative Approaches to Forecasting

- Quantitative methods are based on an analysis of historical data concerning one or more time series.
- A time series is a set of observations measured at successive points in time or over successive periods of time.
- If the historical data used are restricted to past values of the series that we are trying to forecast, the procedure is called a time series method.
- If the historical data used involve other time series that are believed to be related to the time series that we are trying to forecast, the procedure is called a causal method.

Time Series Methods

- Three time series methods are:
  - smoothing
  - trend projection
  - trend projection adjusted for seasonal influence

Components of a Time Series

- The pattern or behavior of the data in a time series has several components.
- The four components we will study are:

Trend	Cyclical	Seasonal	Irregular

Components of a Time Series

- Trend Component
  - The trend component accounts for the gradual shifting of the time series to relatively higher or lower values over a long period of time.
  - Trend is usually the result of long-term factors such as changes in the population, demographics, technology, or consumer preferences.

Components of a Time Series

- Cyclical Component
  - Any regular pattern of sequences of values above and below the trend line lasting more than one year can be attributed to the cyclical component.
  - Usually, this component is due to multiyear cyclical movements in the economy.

Components of a Time Series

- Seasonal Component
  - The seasonal component accounts for regular patterns of variability within certain time periods, such as a year.
  - The variability does not always correspond with the seasons of the year (i.e. winter, spring, summer, fall).
  - There can be, for example, within-week or within-day "seasonal" behavior.

Components of a Time Series

- Irregular Component
  - The irregular component is caused by short-term, unanticipated and non-recurring factors that affect the values of the time series.
  - This component is the residual, or "catch-all," factor that accounts for unexpected data values.
  - It is unpredictable.

Measures of Forecast Accuracy

- Mean Squared Error

  The average of the *squared* forecast errors for the historical data is calculated. The forecasting method or parameter(s) that minimize this mean squared error is then selected.

- Mean Absolute Deviation

  The mean of the *absolute values* of all forecast errors is calculated, and the forecasting method or parameter(s) that minimize this measure is selected. (The mean absolute deviation measure is less sensitive to large forecast errors than the mean squared error measure.)

Smoothing Methods

- In cases in which the time series is fairly stable and has no significant trend, seasonal, or cyclical effects, one can use smoothing methods to average out the irregular component of the time series.
- Three common smoothing methods are:

Moving Averages

Weighted Moving Averages

Exponential Smoothing

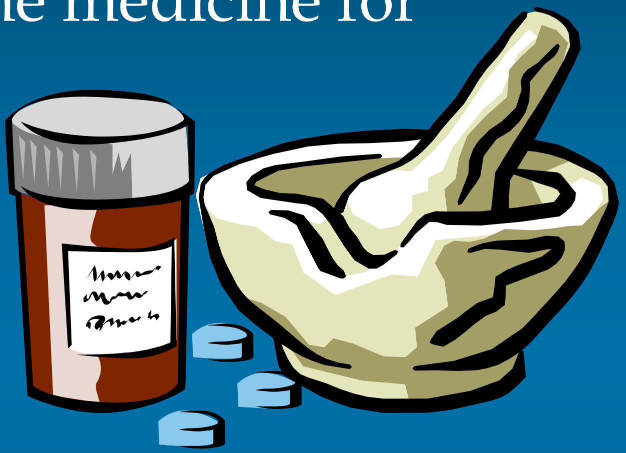
Smoothing Methods

- Moving Averages
  - The moving averages method consists of computing an average of the most recent n data values for the series and using this average for forecasting the value of the time series for the next period.
  - $$\text{Moving Average} = \frac{\sum(\text{most recent } n \text{ data values})}{n}$$

Smoothing Methods: Moving Averages

- Example: Rosco Drugs

▶ Sales of Comfort brand headache medicine for the past ten weeks at Rosco Drugs are shown on the next slide. If Rosco Drugs uses a 3-period moving average to forecast sales, what is the forecast for Week 11?



Smoothing Methods: Moving Averages

- Example: Rosco Drugs

Week	Sales	Week	Sales
1	110	6	120
2	115	7	130
3	125	8	115
4	120	9	110
5	125	10	130

Smoothing Methods: Moving Averages

Week	Sales	3MA	Forecast
1	110		
2	115		
3	125	116.7	
4	120	120.0	116.7
5	125	123.3	120.0
6	120	121.7	123.3
7	130	125.0	121.7
8	115	121.7	125.0
9	110	118.3	121.7
10	130	118.3	118.3
11			$\boxed{118.3}$

$(110 + 115 + 125)/3$

Smoothing Methods

- Weighted Moving Averages
  - To use this method we must first select the number of data values to be included in the average.
  - Next, we must choose the weight for each of the data values.
    - The more recent observations are typically given more weight than older observations.
    - For convenience, the weights usually sum to 1.

Smoothing Methods

- Weighted Moving Averages
  - An example of a 3-period weighted moving average (3WMA) is:

$$3WMA = .2(110) + .3(115) + .5(125) = 119$$

Weights (.2, .3, and .5) sum to 1

Most recent of the three observations

Smoothing Methods

- Exponential Smoothing
  - This method is a special case of a weighted moving averages method; we select only the weight for the most recent observation.
  - The weights for the other data values are computed automatically and become smaller as the observations grow older.
  - The exponential smoothing forecast is a weighted average of *all the observations in the time series*.

Smoothing Methods

- Exponential Smoothing Model

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

To start the calculations, we let $F_1 = Y_1$

where

F_{t+1} = forecast of the time series for period $t + 1$

Y_t = actual value of the time series in period t

F_t = forecast of the time series for period t

α = smoothing constant ($0 \leq \alpha \leq 1$)

Smoothing Methods

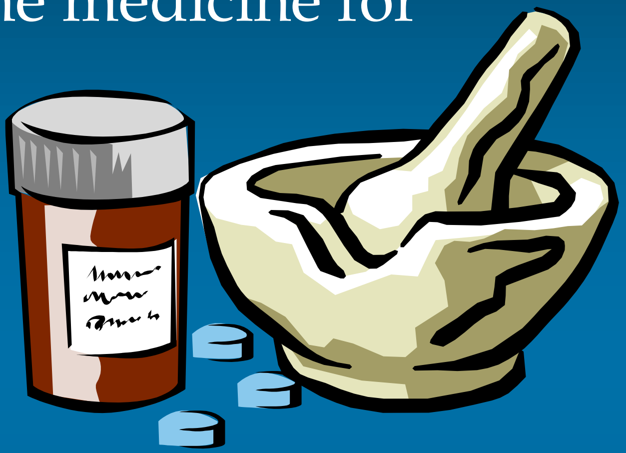
- Exponential Smoothing Model
  - With some algebraic manipulation, we can rewrite $F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$ as:

$$\boxed{F_{t+1} = F_t + \alpha(Y_t - F_t)}$$

  - We see that the new forecast F_{t+1} is equal to the previous forecast F_t plus an adjustment, which is α times the most recent forecast error, $Y_t - F_t$.

Smoothing Methods: Exponential Smoothing

- Example: Rosco Drugs
  - Sales of Comfort brand headache medicine for the past ten weeks at Rosco Drugs are shown on the next slide. If Rosco Drugs uses exponential smoothing to forecast sales, which value for the smoothing constant α, .1 or .8, gives better forecasts?



Smoothing Methods: Exponential Smoothing

- Example: Rosco Drugs

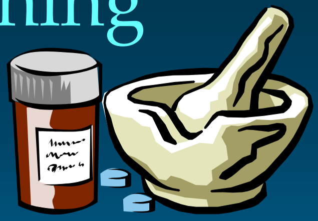
Week	Sales	Week	Sales
1	110	6	120
2	115	7	130
3	125	8	115
4	120	9	110
5	125	10	130

Smoothing Methods: Exponential Smoothing

- Exponential Smoothing (α = .1, 1 - α = .9)

$$
\begin{aligned}
&F_1 && = 110\\
&F_2 = .1Y_1 + .9F_1 = .1(110) + .9(110) && = 110\\
&F_3 = .1Y_2 + .9F_2 = .1(115) + .9(110) && = 110.5\\
&F_4 = .1Y_3 + .9F_3 = .1(125) + .9(110.5) && = 111.95\\
&F_5 = .1Y_4 + .9F_4 = .1(120) + .9(111.95) && = 112.76\\
&F_6 = .1Y_5 + .9F_5 = .1(125) + .9(112.76) && = 113.98\\
&F_7 = .1Y_6 + .9F_6 = .1(120) + .9(113.98) && = 114.58\\
&F_8 = .1Y_7 + .9F_7 = .1(130) + .9(114.58) && = 116.12\\
&F_9 = .1Y_8 + .9F_8 = .1(115) + .9(116.12) && = 116.01\\
&F_{10} = .1Y_9 + .9F_9 = .1(110) + .9(116.01) && = 115.41
\end{aligned}
$$

Smoothing Methods: Exponential Smoothing



- Exponential Smoothing (α = .8, 1 - α = .2)

$$
\begin{aligned}
F_1 &&= 110\\
F_2 &= .8(110) + .2(110) &= 110\\
F_3 &= .8(115) + .2(110) &= 114\\
F_4 &= .8(125) + .2(114) &= 122.80\\
F_5 &= .8(120) + .2(122.80) &= 120.56\\
F_6 &= .8(125) + .2(120.56) &= 124.11\\
F_7 &= .8(120) + .2(124.11) &= 120.82\\
F_8 &= .8(130) + .2(120.82) &= 128.16\\
F_9 &= .8(115) + .2(128.16) &= 117.63\\
F_{10} &= .8(110) + .2(117.63) &= 111.53
\end{aligned}
$$

Smoothing Methods: Exponential Smoothing

- Mean Squared Error

In order to determine which smoothing constant gives the better performance, we calculate, for each, the mean squared error for the nine weeks of forecasts, weeks 2 through 10.

$$[(Y_2\text{-}F_2)^2 + (Y_3\text{-}F_3)^2 + (Y_4\text{-}F_4)^2 + \ldots + (Y_{10}\text{-}F_{10})^2]/9$$

Smoothing Methods: Exponential Smoothing

		$\alpha = .1$		$\alpha = .8$	
Week	Y_t	F_t	$(Y_t - F_t)^2$	F_t	$(Y_t - F_t)^2$
2	115	110.00	25.00	110.00	25.00
3	125	110.50	210.25	114.00	121.00
4	120	111.95	64.80	122.80	7.84
5	125	112.76	149.94	120.56	19.71
6	120	113.98	36.25	124.11	16.91
7	130	114.58	237.73	120.82	84.23
8	115	116.12	1.26	128.16	173.30
9	110	116.01	36.12	117.63	58.26
10	130	115.41	212.87	111.53	341.27
		Sum	974.22	Sum	847.52
	MSE	Sum/9	108.25	Sum/9	94.17

Trend Projection

- If a time series exhibits a linear trend, the method of least squares may be used to determine a trend line (projection) for future forecasts.
- Least squares, also used in regression analysis, determines the unique trend line forecast which minimizes the mean square error between the trend line forecasts and the actual observed values for the time series.
- The independent variable is the time period and the dependent variable is the actual observed value in the time series.

Trend Projection

- Using the method of least squares, the formula for the trend projection is:

$$T_t = b_0 + b_1 t$$

where: T_t = trend forecast for time period t

b_1 = slope of the trend line

b_0 = trend line projection for time 0

Trend Projection

- For the trend projection equation $T_t = b_0 + b_1 t$

$$b_1 = \frac{\sum tY_t - (\sum t \sum Y_t)/n}{\sum t^2 - (\sum t)^2/n}$$

$$b_0 = \overline{Y} - b_1 \overline{t}$$

where: Y_t = observed value of the time series at time period t

$\overline{t}$ = average time period for the n observations

$\overline{Y}$ = average of the observed values for Y_t

Trend Projection

- Example: Auger's Plumbing Service

The number of plumbing repair jobs performed by Auger's Plumbing Service in each of the last nine months is listed on the next slide. Forecast the number of repair jobs Auger's will perform in December using the least squares method.

Trend Projection

- Example: Auger's Plumbing Service

Month	Jobs	Month	Jobs
March	353	August	409
April	387	September	399
May	342	October	412
June	374	November	408
July	396		

Trend Projection

(month) t		Y_t	tY_t	t^2
(Mar.)	1	353	353	1
(Apr.)	2	387	774	4
(May)	3	342	1026	9
(June)	4	374	1496	16
(July)	5	396	1980	25
(Aug.)	6	409	2454	36
(Sep.)	7	399	2793	49
(Oct.)	8	412	3296	64
(Nov.)	9	408	3672	81
Sum	45	3480	17844	285

Trend Projection

- $\bar{t} = 45/9 = 5 \qquad \bar{Y} = 3480/9 = 386.667$
- $b_1 = \dfrac{\sum tY_t - (\sum t \sum Y_t)/n}{\sum t^2 - (\sum t)^2/n} = \dfrac{(9)(17844) - (45)(3480)}{(9)(285) - (45)^2} = 7.4$
- $b_0 = \bar{Y} - b_1\bar{t} = 386.667 - 7.4(5) = 349.667$
- $T_{10} = 349.667 + (7.4)(10) = 423.667$

Trend Projection

- Example: Auger's Plumbing Service

  - Forecast for December (Month 10) using a three-period (n = 3) weighted moving average with weights of .6, .3, and .1 for the newest to oldest data, respectively. Then, compare this Month 10 weighted moving average forecast with the Month 10 trend projection forecast.

Trend Projection

- Three-Month Weighted Moving Average

  The forecast for December will be the weighted average of the preceding three months: September, October, and November.

$$F_{10} = .1Y_{\text{Sep.}} + .3Y_{\text{Oct.}} + .6Y_{\text{Nov.}}$$
$$= .1(399) + .3(412) + .6(408)$$
$$= \underline{\underline{408.3}}$$

- Trend Projection

$$F_{10} = \underline{\underline{423.7}} \text{ (from earlier slide)}$$

Trend Projection

- Conclusion

  Due to the positive trend component in the time series, the trend projection produced a forecast that is more in tune with the trend that exists. The weighted moving average, even with heavy (.6) weight placed on the current period, produced a forecast that is lagging behind the changing data.

Forecasting with Trend and Seasonal Components

Steps of Multiplicative Time Series Model

1. Calculate the centered moving averages (CMAs).
2. Center the CMAs on integer-valued periods.
3. Determine the seasonal and irregular factors ($S_t I_t$).
4. Determine the average seasonal factors.
5. Scale the seasonal factors (S_t).
6. Determine the deseasonalized data.
7. Determine a trend line of the deseasonalized data.
8. Determine the deseasonalized predictions.
9. Take into account the seasonality.

Forecasting with Trend and Seasonal Components

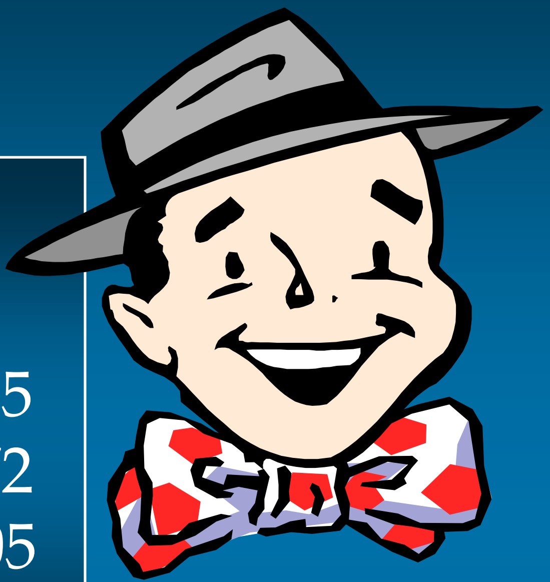
- Example: Terry's Tie Shop

Business at Terry's Tie Shop can be viewed as falling into three distinct seasons: (1) Christmas (November and December); (2) Father's Day (late May to mid June); and (3) all other times. Average weekly sales ($) during each of the three seasons during the past four years are shown on the next slide.

Forecasting with Trend and Seasonal Components

- Example: Terry's Tie Shop

Year	Season 1	Season 2	Season 3
1	1856	2012	985
2	1995	2168	1072
3	2241	2306	1105
4	2280	2408	1120



Determine a forecast for the average weekly sales in year 5 for each of the three seasons.

Forecasting with Trend and Seasonal Components

Step 1. Calculate the centered moving averages.

- There are three distinct seasons in each year. Hence, take a three-season moving average to eliminate seasonal and irregular factors. For example:

1st CMA = (1856 + 2012 + 985)/3 = 1617.67
2nd CMA = (2012 + 985 + 1995)/3 = 1664.00
Etc.

Forecasting with Trend and Seasonal Components

Step 2. Center the CMAs on integer-valued periods.

- The first centered moving average computed in step 1 (1617.67) will be centered on season 2 of year 1. Note that the moving averages from step 1 center themselves on integer-valued periods because *n* is an odd number.

Forecasting with Trend and Seasonal Components

Year	Season	Dollar Sales (Y_t)	Moving Average
1	1	1856	
	2	2012	1617.67
	3	985	1664.00
2	1	1995	1716.00
	2	2168	1745.00
	3	1072	1827.00
3	1	2241	1873.00
	2	2306	1884.00
	3	1105	1897.00
4	1	2280	1931.00
	2	2408	1936.00
	3	1120	

(1856 + 2012 + 985)/3

Forecasting with Trend and Seasonal Components

Step 3. Determine the seasonal & irregular factors ($S_t I_t$).

- Isolate the trend and cyclical components. For each period t, this is given by:

$$S_t I_t = Y_t / (\text{Moving Average for period } t)$$

Forecasting with Trend and Seasonal Components

Year	Season	Dollar Sales (Y_t)	Moving Average	S_tI_t
1	1	1856		
	2	2012	1617.67	1.244
	3	985	1664.00	.592
2	1	1995	1716.00	1.163
	2	2168	1745.00	1.242
	3	1072	1827.00	.587
3	1	2241	1873.00	1.196
	2	2306	1884.00	1.224
	3	1105	1897.00	.582
4	1	2280	1931.00	1.181
	2	2408	1936.00	1.244
	3	1120		

2012/1617.67

Forecasting with Trend and Seasonal Components

Step 4. Determine the average seasonal factors.

- Averaging all $S_t I_t$ values corresponding to that season:

Season 1:	(1.163 + 1.196 + 1.181) /3	= 1.180
Season 2:	(1.244 + 1.242 + 1.224 + 1.244) /4	= 1.238
Season 3:	(.592 + .587 + .582) /3	= .587
		3.005

Forecasting with Trend and Seasonal Components

Step 5. Scale the seasonal factors (S_t).

- Average the seasonal factors = (1.180 + 1.238 + .587)/3 = 1.002. Then, divide each seasonal factor by the average of the seasonal factors.

Season 1: 1.180/1.002 = 1.178

Season 2: 1.238/1.002 = 1.236

Season 3: .587/1.002 = .586

3.000

Forecasting with Trend and Seasonal Components

Year	Season	Dollar Sales (Y_t)	Moving Average	S_tI_t	Scaled S_t
1	1	1856			1.178
	2	2012	1617.67	1.244	1.236
	3	985	1664.00	.592	.586
2	1	1995	1716.00	1.163	1.178
	2	2168	1745.00	1.242	1.236
	3	1072	1827.00	.587	.586
3	1	2241	1873.00	1.196	1.178
	2	2306	1884.00	1.224	1.236
	3	1105	1897.00	.582	.586
4	1	2280	1931.00	1.181	1.178
	2	2408	1936.00	1.244	1.236
	3	1120			.586

Forecasting with Trend and Seasonal Components

Step 6. Determine the deseasonalized data.

- Divide the data point values, Y_t, by S_t.

Forecasting with Trend and Seasonal Components

1856/1.178

Year	Season	Dollar Sales (Y_t)	Moving Average	S_tI_t	Scaled S_t	Y_t/S_t
1	1	1856			1.178	1576
	2	2012	1617.67	1.244	1.236	1628
	3	985	1664.00	.592	.586	1681
2	1	1995	1716.00	1.163	1.178	1694
	2	2168	1745.00	1.242	1.236	1754
	3	1072	1827.00	.587	.586	1829
3	1	2241	1873.00	1.196	1.178	1902
	2	2306	1884.00	1.224	1.236	1866
	3	1105	1897.00	.582	.586	1886
4	1	2280	1931.00	1.181	1.178	1935
	2	2408	1936.00	1.244	1.236	1948
	3	1120			.586	1911

Forecasting with Trend and Seasonal Components

Step 7. Determine a trend line of the deseasonalized data.

- Using the least squares method for t = 1, 2, ..., 12, gives:

$$T_t = 1580.11 + 33.96t$$

Forecasting with Trend and Seasonal Components

Step 8. Determine the deseasonalized predictions.

- Substitute t = 13, 14, and 15 into the least squares equation:

$$T_{13} = 1580.11 + (33.96)(13) = 2022$$
$$T_{14} = 1580.11 + (33.96)(14) = 2056$$
$$T_{15} = 1580.11 + (33.96)(15) = 2090$$

Forecasting with Trend and Seasonal Components

Step 9. Take into account the seasonality.

- Multiply each deseasonalized prediction by its seasonal factor to give the following forecasts for year 5:

Season 1: (1.178)(2022) = 2382
Season 2: (1.236)(2056) = 2541
Season 3: (.586)(2090) = 1225

Qualitative Approaches to Forecasting

- Delphi Approach
  - A panel of experts, each of whom is physically separated from the others and is anonymous, is asked to respond to a sequential series of questionnaires.
  - After each questionnaire, the responses are tabulated and the information and opinions of the entire group are made known to each of the other panel members so that they may revise their previous forecast response.
  - The process continues until some degree of consensus is achieved.

Qualitative Approaches to Forecasting

- Scenario Writing
  - Scenario writing consists of developing a conceptual scenario of the future based on a well defined set of assumptions.
  - After several different scenarios have been developed, the decision maker determines which is most likely to occur in the future and makes decisions accordingly.

Qualitative Approaches to Forecasting

- Subjective or Interactive Approaches
  - These techniques are often used by committees or panels seeking to develop new ideas or solve complex problems.
  - They often involve "brainstorming sessions".
  - It is important in such sessions that any ideas or opinions be permitted to be presented without regard to its relevancy and without fear of criticism.

fcavaibhavjain | vaibhavjainca | vaibhavjain_ca

THANK YOU

CA VAIBHAV JAIN | +91-9711310004

Vaibhav Jain

B.Com(Hons), FCA, ACS, LLB, MBF (ICAI), FAFD (ICAI), Registered Valuer(S&FA), Certified Independent Director (IICA)

Mobile: +91.9711310004 | Email: vaibhavjain@inmacs.com

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